IMAGINARY GEOMETRY OF N.I.LOBACHEVSKY AND IMAGINARY LOGIC OF N.A.VASILIEV

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Abstract

The article deals with conceptual parallels between N.I.Lobachevsky's and N.A.Vasiliev's ideas. The emphasis is made on heuristic prompts connected with Lobachevsky geometry which promoted construction of imaginary logic by N.A.Vasiliev.

The great heuristic significance of non-Euclidian geometry discovery is in undermining the conceptual basis of the idea of Absolutism. Thus the perspectives of the idea of Relativity, plurality of theoretical systems were cleared up and consolitated.

> "Dominating understanding of geometry up to XIX century, - claims H.Mehrtens, - its unicity: one world, one geometry, one truth."[Mehrtens, 1992, p.46].

The possibility of Lobachevsky geometry enables to reason according to analogy and along with classical systems (the icon of which is Euclidian geometry) assume the fact of existence of non-classical systems, to claim namely the fact of their presence. This fact inspired scholars in the quest for non-classical theories.

Surely the way for constructing such theories was thorny, the similar occured to be the way for non-classical logic.

When one tends to describe the landmarks of this way he should take into account that the shifts within classical systems drilling the appearance of non-classical versions have been accomplishing steadily. Steadily and sometimes taking long periods of development which loosen the foundation of classical system, exerted discontent with the latter and, hence, open the possibility of break through to non-classical systems.

The fact that IN PRINCIPLE the alternative and complementary to classical systems are admittable, - the realization of this fact acted as a powerful stimulus in searching of these systems.

Until circa 1880-th there was a widely spread conviction that Aristotelian, classical logic is unique and contained an absolutely complete formulation of the laws of logic. This standpoint is clearly expressed by I.Kant and dominated through almost all XX century (before the mathematical logic was created).

In the Preface to the second edition of the "CRITIQUE OF PURE REASON" Kant is arguing:

> That logic has already, from the earliest times, proceeded upon this sure is evidenced by the fact that since Aristotle it has not required to retrace a single step, unless, indeed, we care to

count as, improvements the removal of certain needless substleties or the clearer exposition of its recognised teaching, features which concern the elegance rather than the certainty of the science. It is remarkable also to the present day this logic has not been able to advance a single step, and is thus to all appearance a closed and completed body of doctrine. If some of the moderns have sought to enlarge it... this could only arise from their ignorance of the peculiar nature of logical science... the sphere of logic is quite precisely delimited [Kant, 1781-7, B VIII - IX].

The same point of view was expressed by the philosophers of science in the early XX century (say, Duhem in 1915).

The irony of history rather often displays itself in the fact that soon after a certain prominent scholars judgement of some domain of science as "complete" and "close" the movements began the result of which is demonstration of openness and incompleteness of this domain of science. That happened with Kant's appraisal of Aristotelian logic state.

The person who did so much to refute the millenium held conviction was N.A.Vasiliev (1880-1940), Russian logician and philosopher, now considered as a forerunner of multi-valued and paraconsistent logics [Bazhanov, 1988, 1990]. In one of works Vasiliev stressed that "Kant himself did his best to refute his own standpoint, concerning logic" [Vasiliev, 1913, p.79].

Soon (certainly, according to historical scale) after Kant wrote his Preface to "CRITIQUE OF PURE REASON" in Aristotelian logic the powerful movement emerged, resulted in eventual drastic changes in logic. Assessing this movement Vasiliev names its following landmarks: Hegel's dialectical logic, Mill's inductive logic and his critical approach towards Aristotelian syllogistic, Sigwart's critique of the classical doctrine of modal judgements and, at last, the development of mathematical logic by Boole, Schroder, Poretsky, Peano, Frege, Russell [Vasiliev, 1912a, 1913, 1924]. Worthy to note that Vasiliev specially stressed "subjective" character of his choice. By the way in Vasiliev's works not were merely mentioned, but thoroughly analysed the works by Poincare and Couturat, Hilbert and de Morgan, Jevons and Venn, C.S.Peirce and W.Hamilton.

The break through the horizon of traditional logic have been taking place in several points. First of all, one of the cornestone laws of Aristotelian logic - the law of contradiction, according to which the existence of judgement (A) and its negation (non-A) in the reasoning is intolerable, i.e. the system should be consistent, - was severely critized by the philosopheres assigned themself to dialectical trend. They were seeking in the world realized contradiction and its reflection in human conscienceness (N.Cusa, Gaman, Hegel, Banzen, Meinong) [Vasiliev, 1913, p.57, 70].

The law of contradiction was subjected to subtle critique by outstanding Polish logician J.Lukasiewicz in 1910.

Lukasiewicz argued that the law of contradiction can't be treated as allegedly proved for its direct evidence for the evidence couldn't stay for criterion of truth. Moreover the law mentioned, Lukasiewicz claimed, never considered as self-evident in the history of science; highly doubtful that the law of contradiction may be viewed as a natural law, determined by physical organization of the human beings or, perhaps, to proof it either through certain definition of affirmation and negation or through definition of false judgements. Profoundly and keenly critizing the law of contradiction, Lukasiewicz in 1910 didn't made an attempt to propose the logic, free of this law. His castigation of the law wasn't reinforced by building up a system, replacing the Aristotelian logic. The law of excluded middle in 1910 was in fact beyond his analysis. Only in 1920 Lukasiewicz put forward three-valued logic which really in certain sense superceded Aristotelian logic.

The deductive method of Aristotelian logic was opposed by inductive doctrine in the works of F.Bacon and J.Mill. However the most vigorous onslaught traditional logic experienced from the mathematical logic, which have been intensively developing since the mid of XIX century. The works of Boole, Peano and especially Frege initiated revolution in logic, resulted in flourishing of mathematical logic.

The emergence of non-classical logic was initiated by all the trends mentioned but the the most noticable contribution was made by mathematical logic supporters. Moreover the notion "non-Aristotelian logic" the most likely emerged in the scope of mathematical logic ideas. Nevertheless the idea of non-Aristotelian logic was pretty vague and uncertain even in the early XX-th century.

The following sample from the article by P.Carus [Carus, 1910, p. 44-46] where the vistas of non-Aristotelian logic creation were discussed is rather typical for the turn of XX-th century (by the way this work was known by Vasiliev soon after it appearance):

> Aristotelian logic is incomplete and insufficient. It treats only the most simple relations and does not cover the more complicated cases of thinking, but so far as it goes it is withiut fault... And why should there not as well exist a curved logic as a mathematics of curved space? A curved logic would be a very original innovation for which no patent has yet been applied for. What a splendid opportunity to acquire Riemann's fame in the domain of logic!... The world has seen many new inventions. Over the telephone we can talk at almost unlimited distances, and some of our contempararies fly like birds through the air. Radium has been discovered which is often assumed with a certain show of plausibility to upset the laws of physics, but the invention of non-Aristotelian logic would cap the climax.

Despite the idea of non-Aristotelian logic in the turn of the XX-th century was pretty blurred, the expectation of its realization seemed to be very promising; the academic climate apparently was ready to cheer the novel logic. None the less the path to discovery of non-Aristotelian logic and its social approval was long and bumpy, more dramatic and complicated than one could foresee.

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The real history of non-Aristotelian logic begins on May 18, 1910 when N.A.Vasiliev presented to Kazan University faculty a lecture "On partial judgements, triangle of opposition, law of excluded forth" [Vasiliev, 1910] to satisfy requirements for obtaining the title of private-dozent, in which were expounded for the first time the key principles of non-Aristotelian, imaginary logic. Thus the birthday of new logic was exactly fixed in the Annals of history. Vasiliev reform of logic was radical; he did his best to test whether it is possible the new logic with new laws, new subject to imply new logical Universe.

Vasiliev began modern non-classical revolution in logic, albeit he certainly didn't complete it. Indeed the revolution was reemerged in the late 1950 - early 1960-th by N. da Costa and D.Nelson for in 1910-th the ideas of Vasiliev were premature to be adopted and to join in.

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One of the main heuristic prompts, the sort of incentive to non-Aristotelian logic of Vasiliev was the the discovery of non-Euclidian geometry by Lobachevsky.

The possibility of "another" logic, distinct from Aristotelian convince us, according to Vasiliev, the existence of another, non-Euclidian geometry.Not only the fact of existence of another geometry inspired the scholar. In geometry itself he has found more than mere prompt.

> "Imaginary logic is constructed by imaginary
> geometry method... In order to implement this method
> I have learned the non-Euclidian geometry... From
> all non-Euclidian geometry systems I have had more
> intently studied the geometry of Lobachevsky, which
> I learned from his original works," - Vasiliev
> stated [Vasiliev, 1911, p.20-21].

In analogy of his logic and Lobachevsky geometry names Vasiliev explored some inward analogies for the logical identity of their creation methods [Vasiliev, 1912a, p.208]. Similar to Lobachevsky geometry starting point was the rejection of attempts to proof the famous V-th postulate and construction of geometry, free of that postulate, the starting point of Vasiliev logic is the abandoning of crucial Aristotelian logic laws - the laws of contradiction and excluded middle - and the construction of logic, free of these laws. Exactly in the bottom unity of methods lies

> "the striking analogies between non-Euclidian
> geometry and... imaginary (non-Aristotelian) logic"
> [Vasiliev, 1912b, p.5].

Both non-Euclidian geometry and non-Aristotelian logic, put it Vasiliev, are sound systems, possible after the giving up of their pivotal statement, both are consistent, both disturbs the common sense and our intuition.

In Euclidian geometry the strict lines on plane surface are either intersect or remain parallel. In Lobachevsky geometry the strict lines on surface are either intersect, or not intersect, or parallel.

In Aristotelian logic we have two different - in relation to their quality - types of judgements, which charactirize the subject-predicate relation - affirmative and negative judgements. In Vasiliev logic there are three classes of judgements, - affirmative, negative and so called "indifferent". Thus

> "the dichotomy of "telluric" logic and geometry transits to trichotomy of imaginary sciences" [Vasiliev, 1912a, p.233; compare: Vasiliev, 1911, p.21].

After almost half of century the existence of Lobachevsky geometry its interpretation on surfaces called pseudosphere was discovered. Imaginary logic, Vasiliev wrote, valid not only in certain imaginary world with two different types of "sensations"; it may be interpreted in "terrestrial" world, in the logic of concepts, which not the same as the "telluric" things logic. Vasiliev demonstrated that in the latter the laws of contradiction and excluded middle are valid, while in logic of concepts we are to adopt the laws of, as he called one, the law of non-selfcontradiction and excluded forth.

The telluric things states might be described by two classes of judgements - affirmative and negative; for the logic of concepts three classes of judgements are required - the affiermative, negative and so called "accidental". The law of excluded forth - the law of imaginary logic - simultaneously is the law of logic of concepts. To "indifferent" class of judgements in imaginary logic correspond class of accidental judgements in the logic of concepts.

> "Imaginary logic may be viewed as realization of logic of concepts, the imaginary world of realized concepts. Plato hypostated the world of ideas; that world should live according the imaginary logic rules,"- stressed Vasiliev [Vasiliev, 1913, p.64].

Pseudosphere is in some sense an ideal construction, but Lobachevsky imaginary geometry in certain physical conditions in the Universe becomes the geometry of real space.

> "Similar to that when our sensational capabilities are organized in particular manner, or the world do the same, the logic must be non-Aristotelian." [Vasiliev, 1912a, p.238].

Our world and sensational abilities are arranged in such a manner that all immediate sensations are positive. "Negative" sensations actually are negative; they are secondary if compared to positive one, and appeares when one feature

replace another, incompatible with the first one. In the world with two kinds of sensations of living beings the non-Aristotelian logic surely reines. To put it in another way the logical laws and principles for the first hand are determine by nature of cognitive objects and experience, open to subject, i.e. they are EMPIRICAL. Arguing the dependance of logical laws origin from some sort of imaginary reality, Vasiliev persistently stressed the primacy of ontological aspect of logic, the thought that material conditions make up various kinds of logic. Changing the ontology, combining the reality features, we can get different imaginary logics for the imaginary logic method opens the possibility to experiment in logic, to give up certain logical principles and to see what we get of this rejection. This method resembles the "natural sciences experimental methods"[Vasiliev, 1913, p.78].

Inspite of apparent inlikeness of logics which could be constructed by Lobachevsky method, all these logics have common feature, invariant for any logic and responsible for the possibility of construction. This common feature manifest in METALOGIC, which contains some logical minimum, independant of diversity of thought contents, but vital for sound reasoning.

Non-Euclidian geometry gives one more crucial lesson for non-Aristotelian logic as well as logic in wide sense. The lesson is that non-Euclidian geometry not only greatly influenced the mathematics ideas flux, but pulled out the importance of the foundational studies. D.Hilbert axiomatized geometry and, hence, cleared up its foundations, the premises of geometrical knowledge used before the studies implicitly. Vasiliev highly appreciated Hilbert's axiomatics and stressed his primacy in the foundational problems:

> "Hilbert showed the remarkable accuracy in the treatment of the question, which might serve like a standard for logic." [Vasiliev, 1911, p.22; 1912a, p.245].

Development of logic reached the stage, said Vasiliev, when the problems of its foundations and axiomatisation should be put into fore. Any logician feels the "chaotic" state of the study of laws and principles of thought, axioms and postulates of logic, belong to the most fundamental issues.

According to Vasiliev imaginary logic method enables to: chose axioms, fundamental for logic and belong to it foundations, give them accurate definitions, to study the axiom interdependance problem, to clear what logical statements and operations depends on certain axioms and, finally, to classify logical axioms. As a result logic might be put in a

> "strongly provable form, similar to that of mathematics" and "logical formulas could be generalised and used in most general style" [Vasiliev, 1913, p.78].

Seeing in mathematics an undisputable standard for logic, Vasiliev doesn't meant the external likeness between them. He was quite learned in contemporary mathematics development (largely due to his father eminent Professor of Mathematics

A.V.Vasiliev [Bazhanov, Iushkevich, 1992). N.A.Vasiliev was informed about the mathematical logic achievements which "have been influencing informal (i.e.Aristotelian - V.B.) logic in decisive even crucial way.

> "Mathematical logic, claimed Vasiliev, can demonstrate the tightest connection between logic and mathematics and to be a powerful tool in foundational studies [Vasiliev, 1913, p.79].

Logic is based, according to Vasiliev, on geometrical intuition. The basic logical relation as in geometry is the relationship between whole and parts of the whole, reduced to the relation betweem foundation and its consequences. Foundation is a whole and consequences its parts. This relation in essence should be assessed as mathematical and it lies in the basis of syllogistic principle.

Logic and mathematics enrich each other. Thats why

> "not only non-Aristotelian logic is an application to logic of non-Euclidian geometry method; we may argue that non-Euclidian geometry is the special case, the application of non-Aristotelian logic method " [Vasiliev, 1911, p.21].

The problem of connection between logic and mathematics Vasiliev seriously discussed with several mathematicians, first of all with well-known in Russia mathematician and geometer N.N.Parfentiev. The result of the discussion was the joint major "The boundary problems of logic and philosophy of mathematics" took place at Kazan University in 1914.

The relationship between logic and mathematics was viewed differently by logicians. Vasiliev distinguished at least two groups: one - "mathematical" - was in favor of tight connection between logic and mathematics, another - "gnoseological" - in favor of tight connection between logic and theory of knowledge and abuses the "formal"(mathematical) logic (B.Croche, W.Windelband).

"What path should logic chose?" - ask the question Vasiliev. Will logic be intensively enriched by mathematical methods or it'll still ignore the success of mathematics? This is the Herculius treshold of logic. Vasiliev was greatly in favor of the first one. In the mathematisation of logic Vasiliev saw the guarantee of logic's bright future.

> "Who could neglect the connection between logic and geometry manifested, for instance, in geometrical diagrams of logic? The possibility of algebraisation of logic clearly shows this relationship"[Vasiliev, 1912c, p.389].

Vasiliev zealously studied mathematics. Moreover, he carefully studied mathematical logic for

> "the mathematical logic can provide the special proof of imaginary logic conceivability"[Vasiliev, 1911, p.24].

At last worthy to note that in 1920-th Vasiliev made an attempt to construct the "mathematical logic of intension" -

contrary to the logic of extension, but this work didn't survived.

Thus Lobachevsky method implenented in imaginary logic build up has deep roots in Vasiliev's position in the foundations of logic.

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